Permutations

المنهج عباره عن

Pure Math. (60 درجه)
- Algebra — 3 Chapters
- Analytic Solid Geometry — 2 Chapters
- Calculus — 4 Chapters

Applied Math. (60 درجه)
- Statics — 6 Chapters
- Dynamics — 4 Chapters



الصورة 1

مصر التعليمية
مباشر

Revision

Permutations

Factorial 9 $\underline{|9} = 9! = 9 \times 8 \times 7 \times \ldots \times 2 \times 1$

$\underline{|9} = 9 \times 8 \times \ldots\ldots \times 2 \times 1$

$\underline{|9} = 9 \times \underline{|8}$

$\underline{|9} = 9 \times 8 \times 7 \ldots\ldots \times 2 \times 1$

$\underline{|9} = 9 \times 8\,\underline{|7}$

$\underline{|10} = 10\,\underline{|9}$

$= 10 \times 9\,\underline{|8}$

$\underline{|n} \quad \underline{|n-1}$

$\underline{|n} = n(n-1)\underline{|n-2}$

$\underline{|n-4} = (n-4)\underline{|n-5}$

$\underline{|n+1} = (n+1).n\,\underline{|n-1}$

$\underline{|6} = 6 \times 5 \times 4 \times \ldots\ldots \times 1$

$\underline{|7.5}$ $\underline{|-5}$

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مدرسة على الهوا

Permutations

Permutations

$$\frac{\lfloor 11}{\lfloor 9} = \frac{11 \times 10 \cancel{\lfloor 9}}{\cancel{\lfloor 9}}$$

$$\frac{\lfloor n-3}{\lfloor n-4} = \frac{(n-3)\cancel{\lfloor n-4}}{\cancel{\lfloor n-4}}$$

$$\lfloor n+3 = 5\lfloor n+2 \Rightarrow (n+3)\cancel{\lfloor n+2} = 5\cancel{\lfloor n+2}$$

$$\lfloor n-1 = 12\lfloor n-3 \Rightarrow (n-1)(n-2)\cancel{\lfloor n-3} = 12\cancel{\lfloor n-3}$$

مدرسة على الهواء

Permutations



N.B.

Permutations

$\lfloor n-2 \Rightarrow n \geq 2$ $\lfloor 3-n \Rightarrow n \leq 3$

Then $n = 2$ or 3

$(n-3)(n-4)=20$ find $\lfloor n$

$n^2 - 7n + 12 - 20 = 0$

$n^2 - 7n - 8 = 0$

$(n-8)(n+1) = 0$

$n=8$, ~~$n=-1$~~

$(n-3)(n-4)=20$ find $\lfloor n$

$(n-3)(n-4) = 5\times4$

$n-3=5$ $n=8$



Permutations

1) Prove that $\lfloor 10 = 2^5 \lfloor 5 \times 9 \times 7 \times \dots \times 1$

$$\lfloor 10 = 10\times9\times8\times7\times6\times5\times4\times3\times2\times1$$
$$=(10 \times 8 \times 6 \times 4 \times 2)\times9\times7\times5\times3\times1$$
$$=(2\times5\times2\times4\times2\times3\times2\times2\times2\times1)\times9\times7\times5\times3\times1$$

$$=2^5 \lfloor 5 \times9\times7\times5\times3\times1$$

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Permutations

Permutations

2) Prove that $\lfloor 100 = 2^{50} \lfloor 50 \times 99 \times 97 \times \ldots \times 1$

$\lfloor 100 = 100 \times 99 \times 98 \times \ldots \times 2 \times 1$

$= (100 \times 98 \times \ldots \times 2) \times 99 \times 97 \times \ldots \times 1$

$= (2 \times 50 \times 2 \times 49 \times \ldots \times 2 \times 1) \times 99 \times 97 \times \ldots \times 1$

$= 2^{50} \lfloor 50 \times 99 \times 97 \times 95 \ldots \times 1$



مدرسة على الهوا

Permutations

$$\underline{|2n} = 2^n \underline{|n} \; 1\times 3\times 5\times \cdots \cdots \times$$

$$\underline{|2n} = 2n(2n-1)(2n-2)\cdots\cdots\; 2\times 1$$

$$= [2n(2n-2)\cdots\cdots 2][1\times 3\times 5\cdots\cdots(2n-1)]$$

$$= 2\times n\cdot 2(n-1)\cdots 2\times 1$$

$$= 2^n \underline{|n} \; 1\times 3\times 5\cdots\cdots(2n-1)$$

Permutations

3) if $n!$, $(n-2)!$, n $(2-n)!$ are the side lengths of a triangle , then the numerical value of the perimeter of the triangle = ...

$(n-2)! \Rightarrow n \geq 2$ and $(2-n)! \Rightarrow n \leq 2$ Then $n=2$

Side lengths are 2,1,2 Perimeter = $2+1+2=5$

4) if $(1+\log x)! = 1$ then $x =$ .... or ....

$(1+\log x)! = 1!$ $1 + \log x = 1 \Rightarrow \log x = 0 \Rightarrow x = 1$

or

$(1+\log x)! = 0!$

$\log_{3} x = 5 \Rightarrow 3^{5} = x$

$\log_{7} (x+1) = 2 \Rightarrow 7^{2} = x+1$

$\log x = 4 \Rightarrow 10^{4} = x$



الصورة 9

Permutations

3) if $|\underline{n}$, $|\underline{n-2}$, $n\,|\underline{2-n}$ are the side lengths of a triangle , then the numerical value of the perimeter of the triangle = ...

$|\underline{n-2} \Rightarrow n \geq 2$ and $|\underline{2-n} \Rightarrow n \leq 2$ Then $n=2$

Side lengths are 2,1,2 Perimeter = 2+1+2=5

4) if $|\underline{1+\log x} = 1$ then $x =$ .... or ....

$|\underline{1+\log x} = |\underline{1}$ $1 + \log x = 1$ $\log x = 0$ $x = 1$

or

$|\underline{1+\log x} = |\underline{0}$ $1+\log x = 0$ $\log x = -1$ $x = \frac{1}{10}$



Permutations

5) $\frac{1}{2}\underline{|n}$, $\underline{|n-2}$, $\underline{|3-n}$ are the side lengths of a triangle , then the numerical value of the Area of the triangle =

$\underline{|n-2}$ ➧ $n \geq 2$ $\underline{|3-n}$ ➧ $n \leq 3$ Then $n = 2$ or 3

if $n = 2$ the 1^{st} side = $\frac{1}{2}\underline{|2} = 1$

the 2^{nd} side = , $\underline{|0} = 1$

the 3^{rd} side = , $\underline{|1} = 1$

if $n = 3$

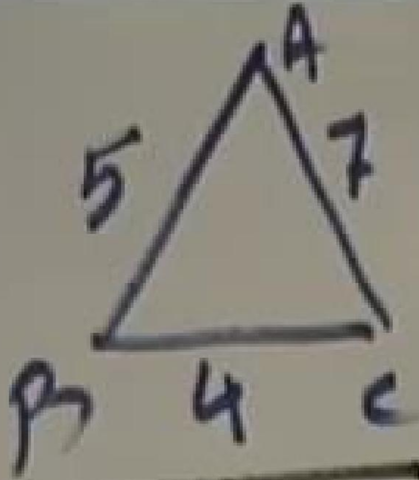
Area of △

① $\frac{1}{2}$ b.h

② $\frac{1}{2}$ Side X Side X Sin included

③ Determinant A-(1,3) B-(4,7) C-(2,9)

④ Heron



$S = \frac{1}{2}(5+7+4)$

$= \frac{1}{2} \times 16 = 8$

$Area = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{8(8-5)(8-7)(8-4)}$

$= \sqrt{8 \times 3 \times 1 \times 4} = \ldots$







الصورة 13



Permutations

5) $\frac{1}{2}\lfloor n$, $\lfloor n-2$, $\lfloor 3-n$ are the side lengths of a triangle , then the numerical value of the Area of the triangle =

$\lfloor n-2 \Rightarrow n \geq 2$ $\quad \lfloor 3-n \Rightarrow n \leq 3$ Then $n = 2$ or 3

if $n = 2$

the 1st side = $\frac{1}{2}\lfloor 2 = 1$ $\quad$ Area = $\frac{1}{2} \times 1 \times 1 \times \sin 60 = \frac{\sqrt{3}}{4}$

the 2nd side = $\lfloor 0 = 1$

the 3rd side = $\lfloor 1 = 1$ $\quad$ Area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{\frac{3}{2}\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-1\right)}$

if $n = 3$

the 1st side = $\frac{1}{2}\lfloor 3 = 3$

the 2nd side = $\lfloor 1 = 1$

the 3rd side = $\lfloor 0 = 1$

$= \sqrt{\frac{3}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{\frac{3}{16}}$







Permutations

5) $\frac{1}{2}\lfloor n$, $\lfloor n-2$, $\lfloor 3-n$ are the side lengths of a triangle , then the numerical value of the Area of the triangle =

$\lfloor n-2 \Rightarrow n \geq 2$ $\quad \lfloor 3-n \Rightarrow n \leq 3$ Then n = 2 or 3

if n = 2

the 1st side = $\frac{1}{2}\lfloor 2 = 1$

the 2nd side = , $\lfloor 0 = 1$

the 3rd side = , $\lfloor 1 = 1$

Area = $\frac{1}{2} \times 1 \times 1 \times \sin 60 = \frac{\sqrt{3}}{4}$

Area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{\frac{3}{2}\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-1\right)}$

$= \sqrt{\frac{3}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{\frac{3}{16}}$

if n = 3 (crossed out)

the 1st side = $\frac{1}{2}\lfloor 3 = 3$

the 2nd side = $\lfloor 1 = 1$

the 3rd side = $\lfloor 0 = 1$

$\lfloor 9 = 9 \lfloor 8$

$7 \lfloor 6 \rightarrow \lfloor 7$

$8 \lfloor 7 \quad \lfloor 8$

$2n \lfloor 2n-1 \rightarrow \lfloor 2n$

$(3n-2) \lfloor 3n-3 \rightarrow \lfloor 3n-2$

$7\lfloor\underline{7} \rightarrow ???$

$(8-1)\lfloor\underline{7} = 8\lfloor\underline{7} - \lfloor\underline{7}$

$= \lfloor\underline{8} - \lfloor\underline{7}$

$5\lfloor\underline{5}$

$(6-1)\lfloor\underline{5} = 6\lfloor\underline{5} - \lfloor\underline{5}$

$= \lfloor\underline{6} - \lfloor\underline{5}$

$$4\,\underline{|4} = \underline{|5} - \underline{|4}$$

$(5-1)$

Prove that:

Permutations

$$1\underline{|1} + 2\underline{|2} + 3\underline{|3} + 4\underline{|4} + \dots\dots (n-1).\underline{|n-1} = \underline{|n} - \underline{|1}$$

N.B.

$$2\underline{|2} = (3-1)\underline{|2} \Rightarrow 3\underline{|2} - \underline{|2} = \underline{|3} - \underline{|2}$$

$$4\underline{|4} = \underline{|4}\,(5-1) = 5\underline{|4} - \underline{|4} = \underline{|5} - \underline{|4}$$

$$7\underline{|7} = \underline{|8} - \underline{|7} \qquad 9\underline{|9} = \underline{|10} - \underline{|9}$$

Permutations

6) Prove that: $1\lfloor 1 + 2\lfloor 2 + 3\lfloor 3 + 4\lfloor 4 + \ldots\ldots(n-1).\lfloor n-1 = \lfloor n - \lfloor 1$

L.H.S. $1\lfloor 1 + 2\lfloor 2 + 3\lfloor 3 + \ldots(n-1)\lfloor n-1$

$(2-1)\lfloor 1 + (3-1)\lfloor 2 + (4-1)\lfloor 3 + \ldots\ (n-1)\lfloor n-1$

$\lfloor 2 - \lfloor 1 + \lfloor 3 - \lfloor 2 + \lfloor 4 - \lfloor 3 + \ldots\ldots\ldots + \lfloor n - \lfloor n-1$

$-\lfloor 1 + \lfloor n = \lfloor n - \lfloor 1$ **R.H.S.**

Permutations

7) Solve $\underline{\lfloor n-2} = 5\,\underline{\lfloor 5n-6}$

$5(ن-1)$

$5ن-5$

$5 \,|\, 5ن-6$

$2ن-6$

$2(ن-3)$

$2 \,|\, 2ن-7$

Permutations

7) Solve $\underline{|n-2} = 5\,\underline{|5n-6}$

$$(n-1)\,\underline{|n-2} = (5n-5)\,\underline{|5n-6}$$

$$\underline{|n-1} = \underline{|5n-5}$$

$n-1=5n-5 \Rightarrow 4n = 4 \Rightarrow$ $\boxed{n = 1}$

8) Solve $(n-3)! = 3(3n-7)!$

$$(n-2)(n-3)! = (3n-6)(3n-7)!$$

$$(n-2)! = (3n-6)!$$

$n-2=3n-6$

Permutations

47 Permutations

47
74

147 Permutations

147
174
714
741
471
417

33 Permutations

(3,7) Permutations
(7,3)

{3,7} Permutations

Conditions

1) Different elements

2) With order



Permutations

$X = \{4,7,8, 9,3\}$

Permutations over set X taken 2 at a time

47, 48, 49, 43 78, 79, 73, 89, 83, 93

74, 84, 94, 34 87, 97, 37, 98, 38, 39

$$^{5}P_{2} = 20$$

$^{5}P_{3}$ Means 5 elements Choose 3 *Different* elements *With order*

$^{7}P_{4}$ Means 7 elements Choose 4 *Different* elements *With order*

~~$^{9}P_{11}$ Means~~







Permutations

$\therefore {}^{7}P_{3} = 7 \times 6 \times 5$

${}^{9}P_{4} = 9 \times 8 \times 7 \times 6$

${}^{20}P_{9} = 20 \times 19 \times 18 \times \dots\dots \times 12$

${}^{30}P_{10} = 30 \times 29 \times 28 \times \dots\dots \times 21$

${}^{n}P_{r} = n \times (n-1) \times (n-2) \dots\dots \times (n-r+1) \quad , \; r \leq n$



9): If the last Factor of ${}^{(2n-5)}P_{11}$ is 21 find the value of n

The last Factor = 21

$2n - 5 - 11 + 1 = 21$

$2n - 15 = 21 \Rightarrow 2n = 36 \Rightarrow n = 18$

النوع الأول

Permutations

With Calculator	Without Calculator	
${}^{x}P_3 = 24$ $\sqrt[3]{24}$ 2.9	${}^{x}P_3 = 24$	2 | 24
${}^{x}P_3 = {}^{4}P_3$ $x = 4$	${}^{x}P_3 = 4 \times 3 \times 2$ $x = 4$	2 | 12 2 | 6 3 | 3
${}^{2x-1}P_2 = 210$ $\sqrt{210}$ 14.4	${}^{2x-1}P_2 = 210$	2 | 210 3 | 105
${}^{2x-1}P_2 = {}^{15}P_2$	${}^{2x-1}P_2 = 15 \times 14$	5 | 35 7 | 7
$2x-1 = 15$		

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Permutations

Permutations

10 Solve ${}^{x}P_3 = 210$ $\sqrt[3]{210}$ 5.94

${}^{x}P_3 = {}^{7}P_3 \Rightarrow$ $x = 7$

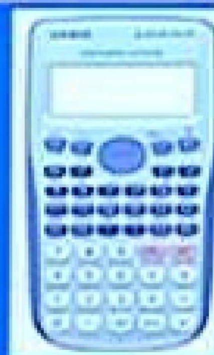


11 ${}^{(x+1)}P_3 = 60$, Find ${}^{x-2}P_2$ $\sqrt[3]{60}$

3.7

${}^{x+1}P_3 = {}^{5}P_3$

$x+1 = 5$ $x = 4$



2	60
2	30
3	15
5	5

${}^{x-2}P_2$ ${}^{2}P_2 = 2 \times 1 = 2$

مدرسة على الهواء

Permutations

Permutations

12 $^{(x-1)}P_4 = 93024$, Find $^{x+1}P_2$

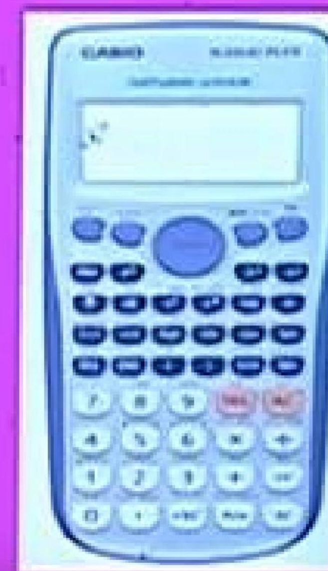
$^{x-1}P_4 = {}^{19}P_4$

$\sqrt[4]{93024}$

17.2

$x - 1 = 19$

$x = 20$



$^{x+1}P_2$

$^{21}P_2 = 21 \times 20 = 420$

Permutations

Permutations

13

$^{10}P_{x+1} = 720$, Find $^{x+3}P_2$

$^{10}P_{x+1} = 10 \times 9 \times 8 = {}^{10}P_3$

$x + 1 = 3$ $x = 2$

$^{x+3}P_2 \Rightarrow {}^{5}P_2 = 5 \times 4 = 20$

Find The middle term of 17 , 30 , 75 , 12 , 20 , 40 , 10

10 , 12 , 17 , 20 , 30 , 40 , 75

The middle term = 20

Permutations

5 , 10 , 15 , 20 , 25 , 30 , 35

The middle term = $\frac{\text{البدايه + النهاية}}{2}$ ➜ $\frac{5 + 35}{2} = 20$

شرطين يا جميل

1) The number is odd

2) The differences is constant

The middle factor Of $^{35}P_{15}$ is …….

35 – 15 + 1

The middle Factor = $\frac{\text{البدايه + النهاية}}{2} = \frac{35 + 21}{2} = 28$

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Permutations

14 If the middle factor Of $^{n}P_{17}$ is 21 Find n

$^{n}P_{17}$ The middle Factor $= \frac{\text{البدايه + النهاية}}{2}$

$$21 = \frac{n + n - 17 + 1}{2}$$

$$\frac{2n - 16}{2} = 21 \Rightarrow 2n - 16 = 42 \Rightarrow 2n = 58 \quad \boxed{n = 29}$$

Permutations

النوع الثانى

$^{n}P_{3} =$

$^{n}P_{3} =$

$^{n}P_{5} =$

$^{n-3}P_{2} =$

$^{7}P_{r-2} =$

Permutations

$$\frac{5x}{5}$$

$$\frac{8\times 7\times 6}{7\times 6}$$

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Permutations



Permutations

النوع الثانى

$$^{n}P_{3} = \frac{n(n-1)(n-2)(n-3)(n-4)\ldots\times 2\times 1}{(n-3)(n-4)\ldots\times 2\times 1}$$

$$^{n}P_{3} = \frac{\underline{|n}}{\underline{|n-3}} \qquad ^{n}P_{5} = \frac{\underline{|n}}{\underline{|n-5}}$$

$$^{n}P_{r} = \frac{\underline{|n}}{\underline{|n-r}}$$

$$^{n-3}P_{2} = \frac{\underline{|n-3}}{\underline{|n-5}} \qquad ^{7}P_{r-2} = \frac{\underline{|7}}{\underline{|9-r}}$$

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$$^{6}P_{r} = 4 \times {}^{6}P_{r-1}$$

$$\frac{6!}{(6-r)!} = 4\,\frac{6!}{(7-r)!}$$

$$\frac{1}{(6-r)!} = \frac{4}{(7-r)\,(6-r)!} \Rightarrow 7-r=4$$

$$\boxed{r=3}$$

Permutations

15

If ${}^6P_r = 4 \times {}^6P_{r-1}$ Find the value of r

$$\frac{6!}{(6-r)!} = 4\,\frac{6!}{(7-r)!}$$

$$\frac{1}{(6-r)!} = \frac{4}{(7-r)\,(6-r)!}$$

$$7 - r = 4 \qquad r = 3$$

Permutations

16

$${}^{n+1}P_5 = 30\ {}^{n-1}P_3$$

$$\frac{(n+1)!}{(n-4)!} = 30\,\frac{(n-1)!}{(n-4)!}$$

$$(n+1)\,n\,(n-1)! = 30\,(n-1)!$$

$$(n+1)n = 6 \times 5$$

$$n = 5$$

Permutations

N.B.

(a, b, c) A.S $\Rightarrow$ $2b = a + c$

$b - a = c - b$

(a, b, c) G.S $\Rightarrow$ $b^2 = a \times c$

$\frac{b}{a} = \frac{c}{b}$

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Permutations

17 If ($^nP_2$, $^nP_3$, $^{n+1}P_3$) Form an A.s Find n

$$2\,{}^np_3 = {}^np_2 + {}^{n+1}p_3$$

$$2\frac{\lfloor n}{\lfloor n-3} = \frac{\lfloor n}{\lfloor n-2} + \frac{\lfloor n+1}{\lfloor n-2}$$

$$2\frac{\lfloor n}{\lfloor n-3} = \frac{\lfloor n}{(n-2)\lfloor n-3} + \frac{(n+1)\lfloor n}{(n-2)\lfloor n-3}$$

$$2 = \frac{1}{(n-2)} + \frac{(n+1)}{(n-2)}$$

$$2 = \frac{n+2}{n-2} \Rightarrow 2n-4 = n+2 \quad n = 6$$

Permutations

18

Prove that ${}^{n}P_{r} = {}^{n-1}P_{r} + r\,{}^{n-1}P_{r-1}$

R.H.S.

$$\frac{\lfloor n-r}{\lfloor n-r-1} + r \,\rule{3cm}{0.4pt}$$

$$\rule{4cm}{0.4pt} + \frac{r}{\rule{4cm}{0.4pt}}$$

L.H.S.

$$R.H.S = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\,\frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-r)\,(n-1)!}{(n-r)\,(n-r-1)!} + \frac{r\,(n-1)!}{(n-r)\,(n-r-1)!}$$



Permutations

$$\frac{(n-1)!}{(n-r-1)!} + r\,\frac{(n-1)!}{(n-r)!}$$

$$\frac{(n-r)\,(n-1)!}{(n-r)\,(n-r-1)!} + \frac{r\,(n-1)!}{(n-r)\,(n-r-1)!}$$

$$\frac{(n-1)!\,(n-r+r)}{(n-r)\,(n-r-1)!} = \frac{n!}{(n-r)!} = {}^{n}P_{r}$$



Permutations

Permutations

18

Prove that ${}^{n}P_{r} = {}^{n-1}P_{r} + r\,{}^{n-1}P_{r-1}$

R.H.S.

$$\frac{\lfloor n-r}{\lfloor n-r-1} + r\,\frac{\lfloor n-1}{\lfloor n-r}$$

$$\frac{(n-r)\lfloor n-1}{(n-r)\lfloor n-r-1} + \frac{r\lfloor n-1}{(n-r)\lfloor n-r-1}$$

$$\frac{\lfloor n-1\,(n-r+r)}{(n-r)\lfloor n-r-1} = \frac{\lfloor n}{\lfloor n-r} = {}^{n}P_{r}$$

L.H.S.



Permutations

Permutations

$$^{n}P_{1} = n = \frac{|\underline{n}}{|\underline{n-1}} = \frac{n\,|\underline{n-1}}{|\underline{n-1}}$$

$$^{n}P_{0} = 1 = \frac{|\underline{n}}{|\underline{n-0}}$$

$$^{4}P_{4} = 4 \times 3 \times 2 \times 1 \quad |\underline{4}$$

$$^{7}P_{7} = |\underline{7}$$

$$^{n}P_{n} = |\underline{n}$$

$$^{n}P_{n} = \frac{|\underline{n}}{|\underline{n-n}}$$

$$|\underline{n} = \frac{|\underline{n}}{|\underline{0}}$$

$$1 = \frac{1}{|\underline{0}}$$

$$|\underline{0} = 1$$



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Permutations

النوع الثالث

Permutations

$$\frac{\lfloor\underline{7}}{\lfloor\underline{3}} = {}^{7}P_{4} \qquad \frac{\lfloor\underline{8}}{\lfloor\underline{6}} = {}^{8}P_{2} \qquad \frac{\lfloor\underline{n+2}}{\lfloor\underline{n-3}} = {}^{n+2}P_{5}$$

$$\lfloor\underline{n+3} = 5\lfloor\underline{n+2} \Rightarrow (n+3)\cancel{\lfloor\underline{n+2}} = 5\cancel{\lfloor\underline{n+2}}$$

$$\lfloor\underline{n-1} = 12\lfloor\underline{n-3} \Rightarrow (n-1)(n-2)\cancel{\lfloor\underline{n-3}} = 12\cancel{\lfloor\underline{n-3}}$$

$$\lfloor\underline{n-2} = 360\lfloor\underline{n-6} \Rightarrow \frac{\lfloor\underline{n-2}}{\lfloor\underline{n-6}} = 360 \Rightarrow {}^{n-2}P_{4} = 360$$



Permutations

Permutations

19 Solve $\lfloor n-3 = 5040 \lfloor n-7$

$\frac{\lfloor n-3}{\lfloor n-7} = 5040 \Rightarrow {}^{n-3}P_4 = 5040$

${}^{n-3}P_4 = {}^{10}P_4$

${}^{x}P_n = {}^{y}P_n$

$x = y$ or

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Permutations

$${}^{x}P_n = {}^{y}P_n \text{ or } \boxed{\begin{matrix} x = y \\ n = 0 \end{matrix}}$$

$${}^{7}P_n = {}^{4}P_n$$

$${}^{7}P \quad {}^{4}P$$



Permutations

N.B.

Permutations

20 Solve $\lfloor n+9 \, \lfloor 6n+1 = \lfloor 5n+1 \, \lfloor 2n+9$

$$\frac{\lfloor 6n+1}{\lfloor 5n+1} = \frac{\lfloor 2n+9}{\lfloor n+9}$$

$${}^{6n+1}P_n = {}^{2n+9}P_n$$

$6n+1 = 2n+9$ Or $n = 0$

$4n = 8$ $n = 2$

Permutations

21 *Solve* $120\,\underline{|2n-3} = \underline{|n-3}\;\underline{|n+5}$

$$\underline{|5}\;\underline{|2n-3} = \underline{|n-3}\;\underline{|n+5}$$

$$\frac{\underline{|2n-3}}{\underline{|n-3}} = \frac{\underline{|n+5}}{\underline{|5}}$$

$${}^{2n-3}P_n = {}^{n+5}P_n$$

$2n - 3 = n + 5$ Or ~~$n = 0$~~

$n = 8$ $\quad$ ($n = 8$)

$\underline{|1} = 1$

$\underline{|2} = 2$

$\underline{|3} = 6$

$\underline{|4} = 24$

$\underline{|5} = 120$

22

Permutations

للشطار بس

$$n!\,(n+2)! = (2n)!$$

$$\cancel{n}\,(n-1)!\,(n+2)! = 2\cancel{n}\,(2n-1)!$$

$$(n-1)!\,(n+2)! = 2!\,(2n-1)!$$

$$\frac{(n+2)!}{2!} = \frac{(2n-1)!}{(n-1)!}$$

$${}^{n+2}P_n = {}^{2n-1}P_n$$

$n+2 = 2n-1$ Or $\cancel{n = 0}$

$n = 3$

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Permutations

Is 11938 *is Divisable By* 46

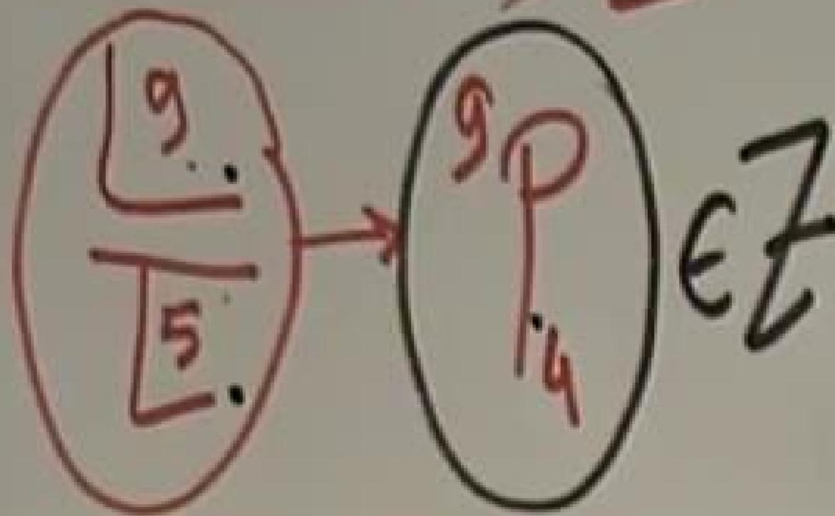
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$$\frac{11938}{46} \in Z \qquad \frac{11938}{46} \notin Z$$

Is $\lfloor 9$ Devisable by $\lfloor 5$

9:36 PM







مدرسة على الهوا

Permutations

E.E.C
مباشر

$$\frac{\underline{|n-1}}{\underline{|n}} = \frac{\underline{|n-1}}{n\,\underline{|n-1}} = \frac{1}{n}$$



مدرسة على الهواء

Permutations

Permutations

N.B. ${}^{x}P_n = {}^{y}P_n$

$x = y$ Or $n = 0$

${}^{n}P_x = {}^{n}P_y$

???

${}^{5}P_4 = 5 \times 4 \times 3 \times 2$

${}^{5}P_5 = 5 \times 4 \times 3 \times 2 \times 1$



Permutations

$$\frac{\lfloor n-1}{\lfloor n} = \frac{\lfloor n-1}{n\lfloor n-1} = \frac{1}{n}$$

$${}^{5}P_{4} = {}^{5}P_{5} \qquad {}^{7}P_{6} = {}^{7}P_{7}$$



Permutations

Permutations

N.B. ${}^{x}P_{n} = {}^{y}P_{n}$ ${}^{n}P_{x} = {}^{n}P_{y}$

???

$x = y$ Or $n = 0$

${}^{5}P_{4} = 5 \times 4 \times 3 \times 2$ ${}^{5}P_{5} = 5 \times 4 \times 3 \times 2 \times 1$

${}^{5}P_{x} = {}^{5}P_{y}$ $x = y$ Or $x = 5$ & $y = 4$ Or $x = 4$ & $y = 5$

(0,0) , (1,1) , (2,2) .... (5,5) (5, 4) (4, 5)



مدرسة على الهوا

Permutations

N.B.

${}^{x}P_{n} = {}^{y}P_{n}$	$x = y$ Or $n = 0$
${}^{n}P_{x} = {}^{n}P_{y}$	$x = y$ Or $x = n$ & $y = n-1$ Or $y = n$ & $x = n-1$



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Permutations

23 Solve $^{20}P_x = {}^{20}P_{x-2y}$

$x = x - 2y$

$0 = -2y \quad y = 0$

$x = x - 0$

$\therefore y = 0 \ \& \ x = 0, 1, 2, \dots 20$

$(0,0), (1,0), (2,0) \dots (20,0)$

Or $x = 20 \ \& \ x - 2y = 19$

$20 - 2y = 19$

$-2y = -1 \quad y = \frac{1}{2}$

$\therefore x = 20 \ \& \ y = \frac{1}{2}$

$(20, \frac{1}{2})$

Or $x = 19 \ \& \ x - 2y = 20$

$19 - 2y = 20$

$-2y = 1 \quad y = -\frac{1}{2}$

$\therefore x = 19 \ \& \ y = -\frac{1}{2}$

$(19, -\frac{1}{2})$

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Summary

$$^{n+1}p_3 = (n+1)\times n\times(n-1)$$

$$^{n+1}p_3 = \frac{\underline{|n+1}}{\underline{|n-2}}$$

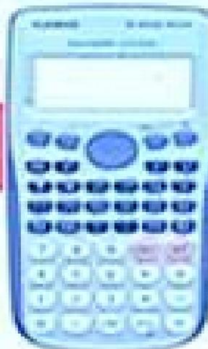
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$$^{n-2}P_3 = 4080$$

$\sqrt[3]{4080}$ 15.6

$$^{n-2}P_3 = {}^{17}P_3$$

$$n-2 = 17 \qquad n = 19$$



II

$$^{n-2}P_5 = 20\ {}^{n-3}P_4$$

$$\frac{\underline{|n-2}}{\underline{|n-7}} = 20\,\frac{\underline{|n-3}}{\underline{|n-7}}$$

$$(n-2)\underline{|n-3} = 20\,\underline{|n-3} \qquad n = 22$$

$^{7}P_{3}$

Meaning — Rule — Value

7 elements choose
3 Different elements
wit order

$$\frac{\underline{|7}}{\underline{|7-3}}$$

$$7\times6\times5$$

III

$$\underline{|n}\ \underline{|3n+3} = \underline{|2n}\ \underline{|2n+3}$$

$$\frac{\underline{|3n+3}}{\underline{|2n+3}} = \frac{\underline{|2n}}{\underline{|n}}$$

$$^{3n+3}P_n = {}^{2n}P_n$$